2000.03.05.19:33 〜

Ono.

Symplectic packing

(Gromov, McDuff-Poltenovich, Biran)

1 $B^n(1)$ の 体積
$S^{n-1}(1)$ 〃

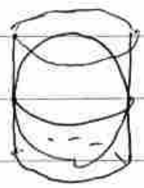
2 $\coprod_{i=1}^{k} B^{2n}(\lambda_i) \underset{symp}{\hookrightarrow} (M^{2n}, \omega) \Leftrightarrow$ M の k 点 blow up 上の symplectic str.

3 proj alg. var. 上の Kähler class はどんなものか
中井の判定法

4. 4次元 symp. mfd $b_2^+ = 1$ etc.
{symplectic class たちの記述

S^1 $\quad 2\pi$

S^2 $\quad 4\pi$



$S^2 \quad \underset{\theta}{S^1} \times \underset{z}{[-1,1]} \quad dz \wedge d\theta$

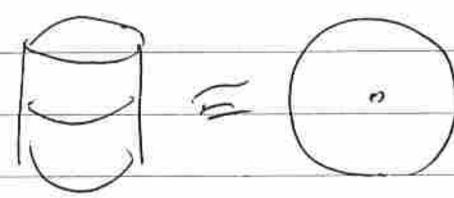
$(0,0,\pm 1)$

$(x,y,z) \mapsto \left(\frac{x}{(x^2+y^2)^{1/2}}, \frac{y}{(x^2+y^2)^{1/2}}, z \right)$ 面積要素を保つ

極座標 (r,θ) $\quad dx \wedge dy = r\,dr \wedge d\theta$

$= d(\frac{1}{2}r^2) \wedge d\theta$

$z = \frac{1}{2} r^2$



$$\begin{array}{ccc} S^3 & S^1 \to & S^3(1) \\ & & \downarrow \\ & & S^2(\frac{1}{2}) \end{array}$$

Hopf fib

$\mathrm{vol}(S^3) = \mathrm{vol}(S^1)\,\mathrm{vol}(S^2(\tfrac{1}{2}))$

$= 2\pi$ $\qquad \cancel{\mathrm{Vol}(\mathbb{CP}^n)}$

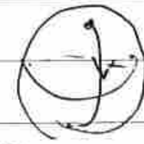
S^{2n+1}

$$\begin{tikzcd} S^1 \arrow[r] & S^{2n+1} \arrow[d] \\ & \mathbb{CP}^1 \subset \mathbb{CP}^n,\ \omega_{F\text{-}S} \end{tikzcd}$$

$$\mathrm{Vol}(\mathbb{CP}^n) = \int \frac{1}{n!} \overbrace{\omega_{F\text{-}S} \wedge \cdots \wedge \omega_{F\text{-}S}}^{n} = \frac{\pi^n}{n!}$$

$$\mathrm{vol}(S^{2n+1}) = \mathrm{vol}(S^1) \times \frac{\pi^n}{n!}$$

S^{2n}



$$\begin{tikzcd} S^{2n-1} \arrow[r] & T_1 S^{2n} \arrow[d] \arrow[loop above, "\varphi_t"] \arrow[r, bend right, "\text{geodesic flow}"'] & \mathrm{Geod}^+(S^{2n}) \\ & S^{2n} & \end{tikzcd}$$

$$\mathrm{Geod}^+(S^{2n}) = \mathrm{Grass}_2(\mathbb{R}^{2n+1}) = Q^{2n-1} \subset \mathbb{CP}^{2n}$$

$$\mathrm{Vol}(T_1 S^{2n}) = \mathrm{vol}(S^{2n-1})\, \mathrm{vol}(S^{2n})$$

$v_1, v_2 \in \mathbb{R}^{2n+1}$ unit vector

$$v_1 + v_2$$
$$v_1 + \sqrt{-1}\, v_2 \in \mathbb{C}^{2n+1}$$
$$(v_1 + \sqrt{-1}\, v_2) \cdot (v_1 + \sqrt{-1}\, v_2)$$
$$= \underbrace{v_1 \cdot v_1 - v_2 v_2}_{=0} + 2\sqrt{-1}\,(\underbrace{v_1 \cdot v_2}_{=0})$$
$$= 0$$

$$\begin{array}{ccc} S^1 \longrightarrow & T_1 S^{2n} & \cancel{\xrightarrow{} T_1S^{2n}/\text{geod flow} \to Q^1 = S^2} \\ & \downarrow & \\ & Q^{2n-1} & \end{array} \qquad \begin{array}{c} \cancel{S^1 \to T_1S^{2n}} \\ \downarrow \\ \cancel{Q^{2n-1}} \end{array}$$

$$\mathrm{Vol}(T_1S^{2n}) = \mathrm{vol}(S^1)\cdot \mathrm{vol}(Q^{2n-1})$$

$$\cancel{\mathrm{vol}(Q^{2n-1}) = \quad}$$

$$\begin{array}{ccc} S^1 \longrightarrow & T_1 S^2 & \xrightarrow{} T_1S^2/\text{geod flow} \quad Q^1 \\ & \downarrow & \quad\quad\quad\quad\quad\quad\quad\quad \| \\ & Q^2 & \quad\quad\quad\quad\quad\quad\quad\quad S^2 \end{array}$$

$$\mathrm{Vol}(T_1S^2) = \mathrm{vol}(S^1)\,\mathrm{vol}(S^2) = 8\pi^2$$

$$\cancel{\therefore \mathrm{Vol}(T_1S^{2n}) =}$$

$$\mathrm{vol}(Q^{2n-1}) = \underline{\qquad\qquad ?}$$

$$\mathrm{vol}\, B^{2n}$$

$$= \int_{B^{2n}} \frac{1}{n!} \overbrace{\omega_0 \wedge \cdots \wedge \omega_0}^{n}$$

$$\omega_0 = \sum dx_i \wedge dy_i$$

Ono

symplectic reduction

symplectic cutting

$$B^{2n} \underset{\text{symp}}{\hookrightarrow} \mathbb{C}P^n$$

open dense

$$(B^{2n} = \mathbb{C}P^n \setminus \mathbb{C}P^{n-1})$$

$$\overline{B^{2n}} \qquad \partial \overline{B}^{2n} = S^{2n-1}$$

$$S^{2n-1} \xrightarrow{\text{Hopf}} \mathbb{C}P^{n-1}$$

$G = S^1 \curvearrowright (M, \omega)$ symp. form

$$(\omega \underset{\text{loc}}{=} \Sigma dx_i \wedge dy_i)$$

ω : S^1-inv

· moment map $\mu : M \longrightarrow \mathbb{R}$ $\quad$ X : S^1作用の inf. action v.f. (infinitesimal)

$$d\mu + i(X)\omega = 0$$

symplectic reduction

$\mu^{-1}(c)/S^1$ "大体" mfd

c μの reg. value

$$\mu^{-1}(c) \overset{i}{\subset} M, \omega$$

$$\downarrow \pi$$

$$M_{?,c} = \mu^{-1}(c)/S^1, \underline{\omega}$$ symp. mfd.

$$\pi^* \underline{\omega} = i^* \omega$$

e.g. $S^{2n-1} \subset \mathbb{C}^n$

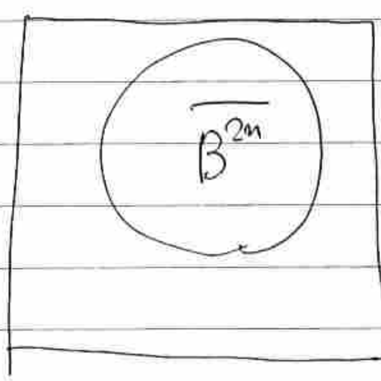
$$\downarrow$$

$$\mathbb{C}P^{n-1}$$

$S^1 \curvearrowright \mathbb{C}^n$ $\quad \lambda(z_1, \cdots, z_n) = (\lambda z_1, \cdots, \lambda z_n)$

$\mu(z_1, \cdots, z_n) = \frac{1}{2}(|z_1|^2 + \cdots + |z_n|^2)$

$\mu(z_1, \cdots, z_n) = c$



$\overline{B^{2n}(1)} = \{ x \in \mathbb{C}^n \mid \mu(x) \leq \frac{1}{2} \}$

$\mathbb{C}^n \setminus B^{2n}(1) = \{$ 〃 $\mid \mu(x) \geq \frac{1}{2} \}$

$\mathbb{C}^n$

$$\overline{B^{2n}(1)} \longrightarrow \mathbb{C}P^n$$

$\mathbb{C}^n \setminus B^{2n}(1) \longrightarrow \mathbb{C}^n$ の1点 blow-up

Ono

$S^1 \curvearrowright \mathbb{C}^n \times \mathbb{C}^{\pm}$

$\lambda \qquad (z_1, \cdots, z_n, w) \mapsto (\lambda z_1, \cdots, \lambda z_n, \lambda w)$ 2通り

$\qquad\qquad (\qquad \text{"} \qquad , \lambda^{-1} w)$

$\mu^{\pm}(z, w) = \frac{1}{2}(\|z\|^2 \pm \|w\|^2)$ $\qquad |z|^2 + |w|^2 = 1$

$\Rightarrow |z|^2 \leq 1$

$(\mu^{\pm})^{-1}(\frac{1}{2}) / S^1 \overset{+}{\supset} B^{2n}(1)$ symp. emb

$\qquad \overset{-}{\supset} \mathbb{C}^n \setminus \overline{B^{2n}(1)}$

$(\mathbb{C}P^2, \underset{m \gg 1}{m\ell})$

$\mathbb{C}P^2$ の1点 blow-up. $m\ell - E$

$(m\ell - E)\cdot(m\ell - E) = m^2 - 1$

$m\ell \cdot m\ell = m^2$

$B^{2n}(\lambda_1) \sqcup B^{2n}(\lambda_2) \xhookrightarrow[\text{symp}]{} B^{2n}(1)$

isometric emb $\quad \lambda_1 + \lambda_2 \leq 1$

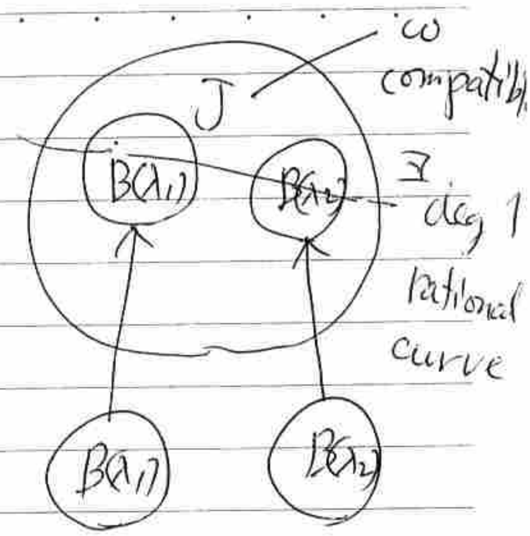
volume preserving emb $\quad \lambda_1^{2n} + \lambda_2^{2n} \leq 1$

symplectic emb $\quad \lambda_1^2 + \lambda_2^2 \leq 1$ (Gromov)

Ono.

$B^{2n}(1)$

$B^{2n}(\lambda_1) \amalg B^{2n}(\lambda_2) \hookrightarrow \mathbb{C}P^{2n}$



toの curve の area

$\vee\!/$

$B(\lambda_1) \amalg B(\lambda_2)$

との intersection の面積 $\Rightarrow \lambda_1^2 + \lambda_2^2 \leq 1$

$\mathbb{C}P^2 \setminus (B(\lambda_1) \amalg B(\lambda_2) \amalg \cdots \amalg B(\lambda_k))$

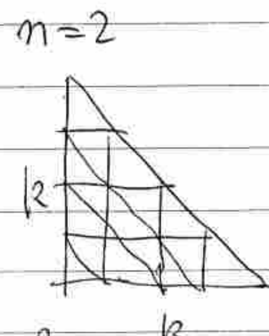
$\downarrow$

$\mathbb{C}P^2$ の 2点 blow-up.　　k点 blow-up.

E_1, E_2　E_k (-1) curves

k	1	2	3	4	5	6	7	8	9	10
obstr		v	v		v	v	v	v		

$\mathbb{C}P^n$　k^n 個の $B^{2n}(\lambda)$ は full volume で disjoint symp にうめこめる.



$\mathbb{C}P^2$, $B^4(1) \subset \mathbb{C}^2$