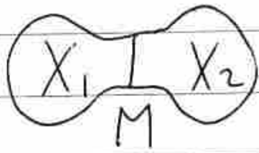
2000.03.05. 16:50〜

Ue

X^4 cl. C^∞ ori. 4-mfd



smooth category

$M \doteq S^3$ $\quad X = \hat{X}_1 \# \hat{X}_2$

$\hat{X}_i = X_i \cup D^4$

$X \# S^4 = X$ 自明な分解.

"irreducible"なものの連結和に
一意的に分かれるか? No.

I_X : X の intersection form $(H_2(X;\mathbb{Z})/\mathrm{Tor}, \cdot)$

$X = X_1 \# X_2 \Rightarrow I_X = I_{X_1} \oplus I_{X_2}$

逆に $I_X = I_1 \oplus I_2$ unimodular symmetric bilinear form

$\Rightarrow X = {}^\exists X_1 \# X_2$? $\quad I_{X_i} = I_i$

No. (D_{inv} SW_{inv} で判定)

X irreducible

$\Longleftrightarrow \quad X = X_1 \# X_2 \Rightarrow X_1$ or $X_2 \underset{h.e.}{\simeq} S^4$

{irreducible 4mfd}

Ue

{irr 4-mfd} ειζ {"minimal" Kähler cpx ~~mf~~ surface}

$\subsetneq$ {symplectic 4 mfd}

$\subsetneq$ {irr. 4 mfd}

e.g. K3 surface X ($b_1 = 0$ $K = 0$)

$\exists\ \infty$ Ja homotopy K3 surfaces

$\exists\ \infty$ symplectic non-complex homotopy K3 (Gompf-Mrowka)

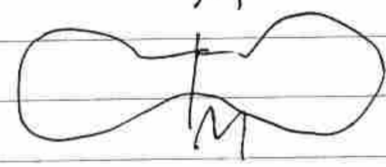
$\exists\ \infty$ irre. non symplectic homotopy K3 (Szabó, Fintushel-Stern)

別の分解 ② Freedman-Taylor

X $\pi_1 X = 1$

$I_X = I_1 \oplus I_2$ uni

$\Rightarrow X = X_1 \underset{\exists M}{\cup} X_2$



homology 3-sphere

<u>Ue</u>

"fiber sum"

$X \supset \Sigma$ ori cl. sf $\Sigma \cdot \Sigma = 0$. とする.

$X' \supset \Sigma$ $\quad N(\Sigma) \cong \Sigma \times D^2$ $\quad \Sigma \cdot \Sigma = 0$.

$X \underset{\Sigma}{\#} X' \quad X \setminus N(\Sigma) \underset{\Sigma \times S^1}{\cup} X' \setminus N(\Sigma)$

$(\varphi \Sigma = 1 \quad \Sigma = T^2)$

?

$X \supset T \cong T^2 \quad T \cdot T = 0 \quad N(T) \cong D^2 \times T^2$

$\varphi : \partial D^2 \times T^2 \xrightarrow[\text{diffeo}]{} \partial (X - N(T))$.

$X_\varphi = (X \setminus N(T)) \underset{\varphi}{\cup} (D^2 \times T^2)$ <u>surgery</u>

"elliptic surface"

E complex surface $E \xrightarrow{\exists \pi} \Sigma$

Rieman surface

general fiber $\pi^{-1}(\text{reg pt}) \approx T^2$

elliptic curve

<u>singular fibers</u>

$E(n) \quad \pi_1 = 1 \qquad \exists \pi : E(n) \to S^2$

$\chi E(n) = 12n$

$\sigma E(n) = -8n \qquad \exists$ cross section

Ue

$E(1) \cong \mathbb{C}P^2 \# 9\overline{\mathbb{C}P^2}$ $\qquad E(1) \supset F$ general fiber

rational elliptic surface $\qquad F \cdot F = 0$

$E(2)$ = K3 surface

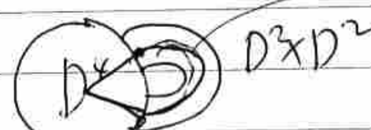
$= E(1) \#_F E(1)$

$E(n) = \underbrace{E(1) \#_F E(1) \# \cdots \#_F E(1)}_{n個}$

F で surgery ⇝ "multiple fiber" をもつ elliptic surf

$E(1) \xrightarrow{\exists \pi} S^2$

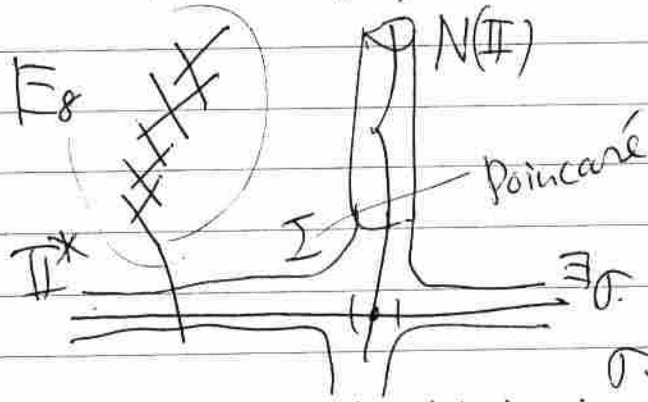
singular fiber として cusp fiber II



$S^3 \supset K$ trefoil knot.

$N(\mathrm{II}) \supset \mathrm{II}$

$E(1) = N(\mathrm{II}) \cup N(\mathrm{II}^*)$



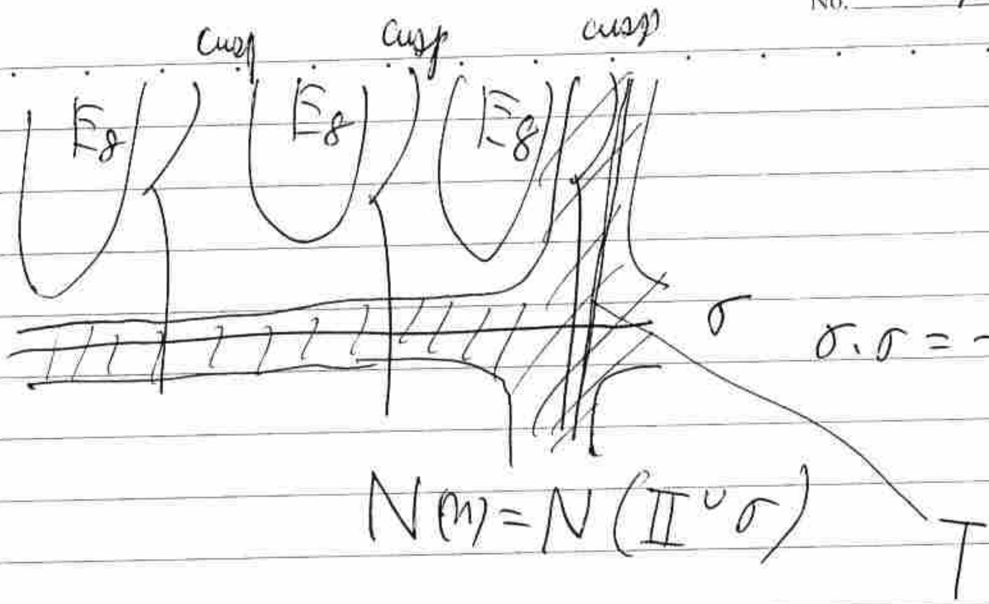
$\exists \sigma$ cross section

$\sigma \cdot \sigma = -1$

$N(1) = N(\mathrm{II} \cup \sigma)$ Nucleus

Ue

$E(n)$



$\sigma \cdot \sigma = -n$

$N(n) = N(\mathrm{II} \cup \sigma)$

一般に $X \supset T \quad T \cdot T = 0$

$$\varphi : \partial D^2 \times T^2 \xrightarrow[\text{diffeo}]{} \underbrace{\partial(X \setminus N(T))}_{\parallel}$$

$T^3 = S^1 \times S^1 \times S^1$

$T^3 = \mathbb{R}^3 / \mathbb{Z}^3$

$a = S^1 \times \cdot \times \cdot$

$b = \cdot \times S^1 \times \cdot$ fix

$c = \cdot \times \cdot \times S^1$

φ の isotopy class

$\varphi(\partial D^2 \times \cdot \times \cdot) = pa + qb + rc \quad \gcd(p, q, r) = 1$

$\varphi(\cdot \times S^1 \times \cdot) =$ 〃

$\varphi(\cdot \times \cdot \times S^1) =$ 〃

できまる.

X_φ の diffeo type (p, q, r) できまる.

$\parallel$

$X \setminus N(T) \cup_\varphi D^2 \times T^2$

さらに $X \supset N(\mathrm{II}) \supset T_{\text{general fiber}}$

X_φ の diffeo type は p のみで決まる

$N(\mathrm{II})$

X と X_φ の SW inv の surgery formula

Morgan Mrowka Szabó

(明日やる.)