2000.03.06. 14:45〜

Ue

$\tilde{P} \to X$ spin^c str $L = \det \tilde{P}$ $c_1 L \in H^2(X, \mathbb{Z})$

$$c_1 L \in H^2(X,\mathbb{Z}) \cong_{P.D.} H_2(X,\mathbb{Z}) \ni L$$

$\boxed{b_2^+ > 1}$ $\left(\begin{matrix} H_1(X,\mathbb{Z}) \\ \text{2-torsion なし} \end{matrix} \Rightarrow \begin{matrix} \tilde{P} \to L \\ \text{one to one} \end{matrix} \right)$

$$\mathcal{A} = \mathcal{A}(L) \times \Gamma(S^+) \curvearrowleft \mathcal{G}_X = Map(X, U(1))$$

$$\mathcal{A} \supset Sol = \{ SW \text{ equation} = 0 \}$$

$$\mathcal{B} = \mathcal{A}/\mathcal{G}_X \supset \mathcal{M}_X(\underset{L}{\tilde{P}}) = Sol/\mathcal{G}_X$$

$$\dim \mathcal{M}_X = \frac{1}{4}\left(c_1 L^2 - (2\chi + 3\sigma)\right) = d_{\tilde{P}}$$

$$d_{\tilde{P}} = 0 \Longrightarrow SW_X(\tilde{P}) = \# \mathcal{M}_X(\tilde{P})$$

$d_{\tilde{P}} > 0$

$$\begin{array}{ccc} \mathbb{L} & \text{universal line bdle} & \\ \downarrow & & \{g|_x = id\} \\ \mathcal{B}^* \times X & & \| \\ \text{irr} & & \mathcal{A}^*/\mathcal{G}_X^0 \\ \mathbb{L} & \supset \mathbb{L} \curvearrowleft \exists s \text{ cross section} & \downarrow S^1 \\ \downarrow & & \mathcal{A}^*/\mathcal{G}_X \\ \mathcal{B}^* \times \{x\} \supset \mathcal{M} \times \{x\} & & \end{array}$$

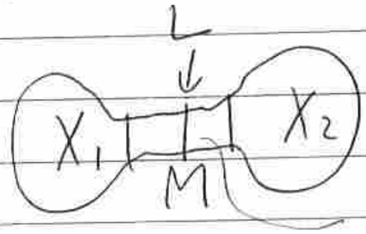
$$SW_X(\tilde{P}) = \langle c_1(\mathbb{L})^{\frac{d_{\tilde{P}}}{2}}, [\mathcal{M}_X \tilde{P}] \rangle$$

$d_{\tilde{P}}$ odd $\to SW_X(\tilde{P}) = 0$ とおく

$$= \# \bigcap \cap s^{-1}(0) \cap [\mathcal{M}_X \tilde{P}]$$

Ue

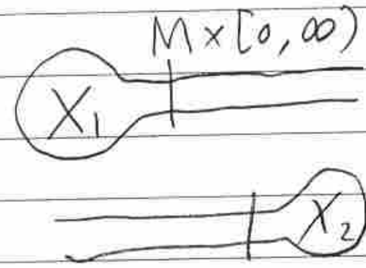
L が SW basic class $\Leftrightarrow$ $SW_X(L) \neq 0$



$L|_{X_1}$ $L|_{X_2}$ $L|_M$ L

product metric on $M \times [R\ R]$

$\mathcal{M}_X(L)$



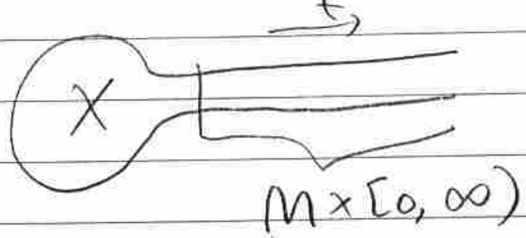
$\mathcal{M}_{X_1}(L_1)$

$\mathcal{M}_{X_2}(L_2)$

$\mathcal{M}_M(L|_M)$

L_1

$\downarrow$



$M \times [0,\infty)$ 上の SW-equation (A, ϕ)

$\mathcal{A}_M = \mathcal{A}(L|_M) \times \Gamma(S_M)$

$S^+|_{M\times\{t\}} \cong S_M \cong S^-|_{M\times\{t\}}$ $\quad$ Spin$^c(3)$ str on M

$$\begin{tikzcd} {}^{\exists}f : \mathcal{A}_M \arrow[r] \arrow[d] & \mathbb{R} \arrow[d] \\ \bar{f} : \mathcal{A}_M/\mathcal{G}_M \arrow[r] \arrow[ur] & \mathbb{R}/\mathbb{Z} \end{tikzcd}$$

Chern-Simon-Dirac function

Ue

temporal gauge $(A(t), \phi(t)) \in \mathcal{A}_{M\times}$ 沿う

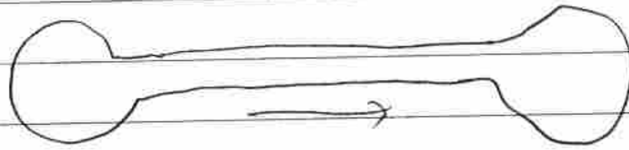
$$(A,\phi) \longleftrightarrow \frac{d}{dt}(A(t),\phi(t)) = -\underbrace{\operatorname{grad} f(A(t),\phi(t))}_{\parallel}$$

$M_{X_1}(L_1) = \{ sol : \int |F_A|^2 < \infty \} / \mathcal{G}_{X_1}$ $\quad (*(F_A - (\phi^* \otimes \phi)_0), D_A\phi)$

$M_M(L_M) = \operatorname{Crit} \bar{f}$ $\quad = 0$

$D_A : \Gamma(S) \to \Gamma(S)$

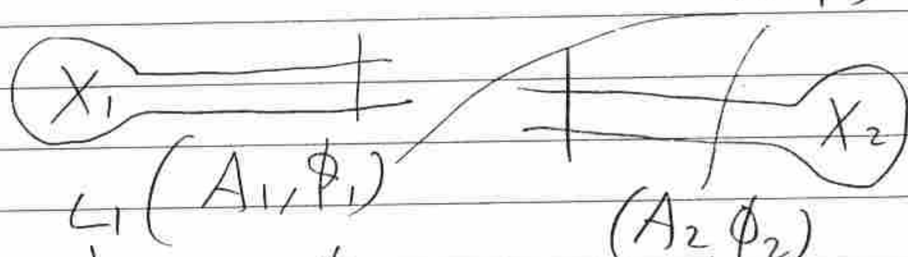
Dirac Operator



$M_X(L)$

$\downarrow$

(A,ϕ)



もし $c_1 L|_M$ torsion ならば

energy finite

$\mathbb{R}\times M$ 上の SW eq $\longmapsto$ length finite

static 解の [illegible] on M

Ue

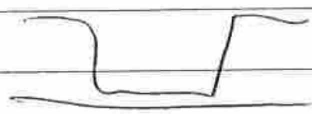
ℓ_i non deg
(0-dim)

X₁にのびる
M a gauge grp

$$\begin{tikzcd} \exists \partial_i \quad \mathcal{M}_{X_i}(L_i) \arrow[r] & Sol_M / \mathcal{G}_{M,X_i} \arrow[d] \\ & \mathcal{M}_M L_M \end{tikzcd}$$

cuper i^* $H^1(X_1, \mathbb{Z}) \to H^1(M, \mathbb{Z})$
?

gluing



$$\begin{tikzcd} \mathcal{M}_{X_1}(L_1) \arrow[r, "\partial_1"] & Sol_M / \mathcal{G}_{M,X_1} \arrow[dr] & & Sol_M / \mathcal{G}_{M,X_2} \arrow[dl] & \mathcal{M}_{X_2}(L_2) \arrow[l, "\partial_2"'] \\ (A_1 \phi_1) \arrow[rr] & & Sol_M & & (A_2 \phi_2) \arrow[ll] \end{tikzcd}$$

deformation complex at $(A_1 \phi_1)$

$$\Omega^0(X_1; \mathbb{R}) \xrightarrow[\text{gauge tr}]{\text{infinitesimal}} \Omega^1(X_1; \mathbb{R}) \times \Gamma(S^+_{X_1}) \xrightarrow[\text{SW a linearization}]{} \Omega^+(X_2; \mathbb{R}) \times \Gamma(S^-_{X_1})$$

$$H^1_{(A_1 \phi_1)} \cong T_{(A_1, \phi_1)} \mathcal{M}_{X_1}(L_1)$$

$$H^2_{(A_1, \phi_1)} = 0 \quad H^2_{(A_2, \phi_2)} = 0$$

ならば真の解がえられる.

Ue

$$\coprod M_X(L) = M_{X_1}(L_1)\times M_{X_2}(L_2)$$

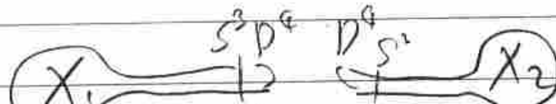
$L|_{X_1}=L_1$ $\qquad M_n(L)$

$L|_{X_2}=L_2$

$X = X_1 \# X_2 \quad b^+(X_1)>0$

trivial

$M_{S^3}(L_{S^3}) = \{(0,0)\}$



$(A_1,\psi_1) \quad (A_2,\phi_2)$

$\dim M_X(L)=0$

$\dim M_{X_1\setminus D^4}(L) = \dim M_{X_1}(L)$

$\dim M_X(L) = \dim M_{X_1}(L) + \dim M_{X_2}(L) + 1$

$X = X_1 \# X_2 \quad b^+X_1>0$

$b^+X_2=0$

reducible solの寄与

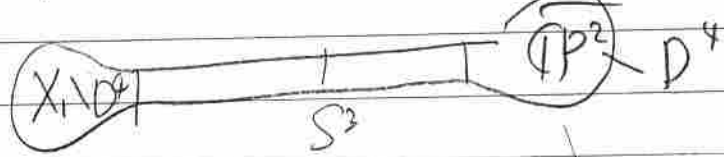
blow up formula $X = X_1 \# \overline{\mathbb{C}P^2} \supset E$ exceptional curve

$L = L_1 + aE \equiv w_2(X)\ (2)$, a odd

$\dim M_X(L) = \dim M_X(L) - \frac{a^2-1}{4} \geq 0$

$\Rightarrow SW_X = SW_{X_1}$

Ue



$(\theta, 0)$ $\quad \exists^1 (A, 0)$

$\underline{F_A^+ = 0}$

$\mathcal{M}_{\overline{\mathbb{C}P^2}}(aE)$ $\quad H^2_{(A,0)} \cong \operatorname{Coker} D_A \cong \mathbb{C}^{\exists d}$

$\widetilde{\mathcal{M}}_{X_1}(L) \times \mathbb{C}^{d'}$

$$\begin{array}{ccc} \| & & \mathcal{G}/\mathcal{G}^0 \\ Sol_{X_1}/\mathcal{G}^0 & \exists s & \| \\ \downarrow & & S^1\text{-equiv section} \\ \widetilde{\mathcal{M}}_{X_1}(L) & & \end{array}$$

$\mathcal{M}_X(L) = \rho^{-1}(0)/S^1$

$\| $

$(c_1 \mathbb{L})^{d'} \cap \mathcal{M}_1(L)$

$$\begin{array}{c} (Sol_{X_1})/\mathcal{G}^0 \\ \downarrow S^1 \\ (Sol_{X_1})/\mathcal{G} \end{array}$$

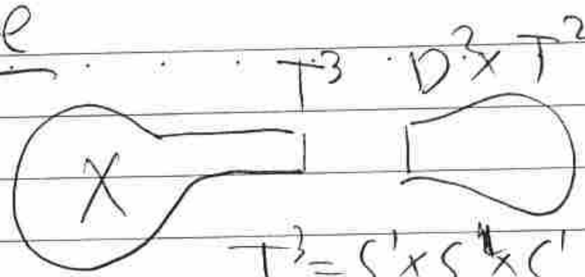
$SW_X(L + aE) = \langle c_1 L^{\exists d}, c_1 \mathbb{L}^{d'} \cap M_X L \rangle$

$= SW_{X_1} L_{X_1 \# \overline{\mathbb{C}P^2}}$

X_1 simple $\quad X_1$ a basic class

$a = \pm 1 \quad L_1 \longleftrightarrow L_1 \pm E$

Ue



$T^3 = S^1 \times S^1 \times S^1$

$\quad\quad\;\; a \quad\;\; b \quad\;\; c$

$\gamma = pa + qb + rc \quad \gcd(p,q,r)=1$

$X_\gamma = X \underset{\varphi(\partial D^2)=r}{\cup} D^2 \times T^2$

Morgan–Mrowka–Szabó

$L_r \to X$ $\quad X_r$ の spin^c str

$L_r|_{D^2\times T^2}$ nontrivial

$\Longrightarrow \; SW_{X_r}(L_r) = 0$

$L_0 \to X$ spin^c str $L_0|_{\partial X}$ trivial

$\Rightarrow$

$$\sum_{L_r|_X = L_0} SW_{X_r}(L_r) = p \sum_{L_a|_X = L_0} SW_{X_a}(L_a) + q \sum_{L_b|_X = L_0} SW_{X_b}(L_b) + r \sum_{L_c|_X = L_0} SW_{X_c}(L_c)$$

Ue

$X_a \supset {}^{\exists}$ nbd $\supset T_a$ general fiber

$D^2 \times T^2 \supset \cdot \times T^2$ $\quad H_1 \partial X \xrightarrow{0} H_1 X$

$\qquad\qquad\quad \overset{\shortparallel}{T_a}$

$\boxed{p \neq 0}$ $\quad SW_X \overset{\text{def}}{=} \sum_{L : X\text{ の basic class}} SW_X(L) e^L$

$\Longrightarrow$

$$SW_{X_r} = SW_X \frac{\sinh T_a}{\sinh T_r}$$

$[T_a] = p[T_r]$ on X_r