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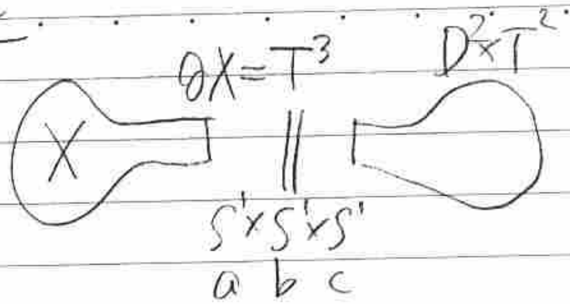
Ue



$b^+ > 1$ $\quad \gamma = pa + qb + rc \sim \partial D^2 \times \cdot \times \cdot$

$$X_\gamma = X \cup_\gamma D^2 \times T^2$$

ThA $\displaystyle \sum SW_{X_\gamma}(L_\gamma) = p \sum_{L_a|X = L_0} SW_{X_a}(L_a) + q \sum_{L_b|X = L_0} SW_{X_b}(L_b) + r \sum_{L_c|X = L_0} SW_{X_c}(L_c)$

$\Big($ $L_0 \to X$ Spin^c str
$L_0|_{\partial X}$ trivial (det line bundle)
L_γ X_γ Spin^c str
$L_\gamma|_{D^2 \times T^2}$ nontrivial
$\Rightarrow SW_{X_\gamma} L_\gamma = 0$ $\Big)$

特に $H_1(\partial X, \mathbb{Z}) \xrightarrow{0} H_1(X, \mathbb{Z})$

$X_a \supset \exists$ cusp nbd $\supset \underline{T_a}$

$$SW_X := \sum_{L : X_a \text{ basic class}} SW_X(L) e^L$$

ThB ($p \neq 0$) $\displaystyle SW_{X_\gamma} = SW_{X_a} \frac{\sinh T_a}{\sinh T_\gamma} = SW_{X_a} \frac{e^{T_a} - e^{-T_a}}{e^{T_\gamma} - e^{-T_\gamma}}$

$$= SW_{X_a} \frac{e^{pT_\gamma} - e^{-pT_\gamma}}{e^{T_\gamma} - e^{-T_\gamma}} = SW_{X_a}\left(e^{(p-1)T_\gamma} + e^{(p-3)T_\gamma} + \cdots + e^{-(p-1)T_\gamma}\right)$$

$T_a = pT_\gamma$ in $H_2(X_\gamma)$ $\rightsquigarrow$ $\beta + (p-1-2a)T_\gamma$: X_γ の basic
$\quad a = 0 \cdots p$

β : X_a の basic class

$E(2) = $ K3 surface

$SW_{E(2)} = 1$

$\cup$

$F = T_a$

$$SW_{E(2)_p} = \frac{e^{pT_r} - e^{-pT_r}}{e^{T_r} - e^{-T_r}}$$

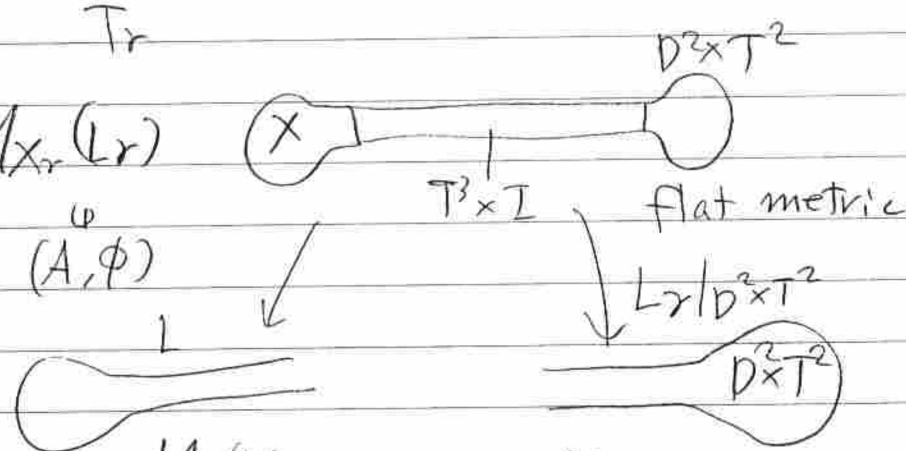
Kähler

T_r

$\mathcal{M}_{X_r}(L_r)$

$\ni$

(A, ϕ)



$\mathcal{M}_X(L) \ni (A_1, \phi_1)$

$\mathcal{M}_{D^2\times T^2}(L_r|_{D^2\times T^2}) \ni (A_2, \phi_2)$

$$(A_1,\phi_1) \searrow \mathcal{M}_{T^3}(L|_{T^3}) \swarrow (A_2,\phi_2)$$

$$\begin{array}{ccc} \mathcal{A}_{T^3} = \mathcal{A}(L_{T^3}) \times \Gamma(S_{T^3}) & \xrightarrow{f} & \mathbb{R} \\ \downarrow & \nearrow_{f} & \\ \mathcal{A}_{T^3}/\mathcal{G}_{T^3} & & \end{array}$$

cylinder 上の SWeq $\Longleftrightarrow$ $\dfrac{\partial}{\partial t}(A(t), \phi(t)) = -\mathrm{grad}\, f(A(t), \phi(t))$

temporal gauge

$$= \begin{pmatrix} *(F_{A_t} - (\phi_t \otimes \phi_t^*)_0 \\ D_{A(t)} \end{pmatrix}.$$

<u>Ue</u>

$\exists (A_\infty, \phi_\infty)$ gradient flow

$\qquad \parallel$

$\lim_{t\to\infty}(A(t), \phi_t) \in \mathrm{Crt}\, f$ に近づく

$\phi_\infty = 0 \quad F_{A_\infty} = 0$

$\mathcal{M}^0_{T^3}(L_T)$

${}^\exists \partial_\infty \; \mathcal{M}_X(L) \longrightarrow \mathcal{S}ol^3/\mathcal{G}_{T^3,X}$ ← Xにのびる T^3 の gauge

$\mathcal{M}_X(L) = Sol/\mathcal{G}_X$

$\bar\partial_\infty \quad \mathbb{R}^3$

$\mathcal{S}ol^3/\mathcal{G}_{T^3,X} \downarrow \mathrm{coker}\, i^*(H^1(X) \to H^1 T^3)$

$\mathbb{R}^3 \to Sol^3/\mathcal{G}_{T^3} = \mathcal{M}_{T^3}(L_T)$

$\downarrow$

T^3

$\{(A_0, 0)\}$

flat conn

$\mathcal{M}_{T^3}(L|_{T^3}) \subset \mathcal{A}_{T^3}/\mathcal{G}_{T^3}$

$$0 \to \Omega^0(i\mathbb{R}) \to \Omega^1(i\mathbb{R}) \times \Gamma(S_{T^3}) \to \Omega^1(i\mathbb{R}) \times \Gamma(S_{T^3}) \to 0$$

$$H^1_{(A_\infty,0)} \cong H^1(T^3; i\mathbb{R}) + \mathrm{Ker}\, D_{A_\infty}$$

$$\mathrm{Ker}\, D_{A_\infty} = \begin{cases} 0 & A_\infty \neq \theta \\ \mathbb{C}^2 & A_\infty = \theta \end{cases}$$ ← trivial conn

$\mathcal{M}_{T^3}(L|_T) \ni \theta$

$\mathcal{M}_{T^3}(L|_T) \setminus \partial_\infty^{-1}(\theta)$

$$\dim(\;\;{}''\;\;) = \frac{1}{4}\Big(c_1 L^2 - \big(\underbrace{2\chi(X) + 3\sigma(X)}_{=0}\big)\Big)$$

A-P-S の index th

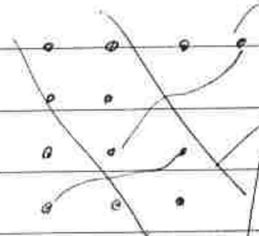
$\partial_\infty^{-1}(\theta)$ の近く

$\mathcal{H}'_{(A_\infty,0)}$ ← f の flow, grad flow

本来の flow は近似でできる.

Ker $D_\theta \neq 0$ ST^3 trivial holonomy の conn

${}'\mathcal{M}_X(L) \xrightarrow{\partial_\infty}$



$\cup$

$\partial_\infty^{-1}(W_\gamma)$

∂'_∞ の image hypersurface

W_γ wall

$$\frac{Sol_{D^2\times T^2}}{\mathcal{G}_{D^2\times T^2,\gamma}} \qquad \downarrow \qquad \mathcal{M}_{T^3}(L|T^3) \ni \theta$$

$$\downarrow \partial'_\infty \qquad \searrow$$

$$\begin{array}{ccccc} \mathcal{M}_X(L) & \xrightarrow{\partial_\infty} & \mathcal{M}^0_{T^3}(L) & & \mathcal{M}_{D^2\times T^2}(L) \quad \partial D^2=\gamma \\ & \searrow^{\bar\partial_\infty} & \downarrow & \swarrow_{\bar\partial_\infty} & \\ & & \mathcal{M}_{T^3}(L) & & \end{array}$$

$(A,0)$ flat

$$\#\mathcal{M}_X(L)\cap\partial_\infty^{-1}W_\gamma = p\,\#\mathcal{M}_X(L)\cap\partial_\infty^{-1}W_a + q\,\#\mathcal{M}_X(L)\cap\partial_\infty^{-1}W_b$$
$$+\, r\,\#\mathcal{M}_X(L)\cap\partial_\infty^{-1}W_c$$

Ue

$$\begin{tikzcd} H^2(X,\partial X) \arrow[r] & H^2X \arrow[r] & H^2\partial X \\ L' \arrow[r, mapsto] \arrow[d] & L \arrow[d, "\mathrm{mod}\ 2"] & \\ W_2(X,\partial X) & W_2 & \end{tikzcd}$$

$\delta: H^2(X,\partial X) \to H^2(X_\gamma)$
$\| \hspace{2em} \nearrow$
$H^2(X_\gamma, D^2\times T^2)$

Th′ $SW_{X_\gamma} = \left(\sum_{L'} SW_X(L')e^{\delta(L')}\right)(e^{T_\gamma} - e^{-T_\gamma})^{-1}$

$SW_X(L') = \# M_X(L) \cap \partial_\infty^{-1}(\theta')$

$$\begin{tikzcd} & H_1T^3 \arrow[dl] \arrow[dr] & \\ H_2X \oplus H_2(D^2\times T^2) \arrow[d, "T_\gamma\ \beta"] & & \\ H_2X_\gamma \arrow[d] & & \\ H_1T^3 & & \end{tikzcd}$$

$H_2(D^2\times T^2) = [T_\gamma]$

$\overset{P.D.}{\approx}$ $H_2(X,\partial X) \oplus H_2(D^2\times T^2_\gamma) \to H^2(X_\gamma)$

$\delta_1: H^2(X,\partial) \to H^2X_\gamma$ $\quad \beta = \beta' + aT_\gamma$

$SW_{X_\gamma} = \left(\sum_{L'} SW(L')e^{\delta_1(L')}\right)(e^{T_\gamma} - e^{-T_\gamma})^{-1}$

$SW_{X_a} = \sum SW(L')e^{\delta_2(L')}(e^{T_a} - e^{-T_a})^{-1}$

$\delta_2: H^2(X,\partial) \to H^2X_a$

map の存在 $\Downarrow$

$SW\,L' \neq 0$ なる L' に対して $\delta_1\ \delta_2$ one to one

Ue

Fintushel-Stern の構成

$X^4 \supset^{\exists}$ cusp nhd $\supset^{\exists} T$ $\quad T \cdot T = 0$
$b^+ > 1$ $\qquad$ torus

$\pi_1(X \setminus \mathrm{cusp}) = 0$

X

$K \subset S^3$ knot

$$X_K = (X \setminus N(T)) \cup (S^3 \setminus N(K)) \times S^1$$

$X \setminus N(T)$: $\cong D^2 \times T$, $\partial D^2 \longleftrightarrow l$, $= S^1 \longleftrightarrow m$

$\begin{pmatrix} D^2 \times S^1 \text{をぬいて} \\ S^3 \setminus N(K) \text{を入れる} \end{pmatrix} \times S^1$

Th $\quad SW_{X_K} = SW_X \, \Delta_K(e^{2T})$

$\Delta_K(t) = a_0 + \sum_{i=1} a_i (t^i + t^{-i})$ $\quad$ K の Alexander poly

$$\left(X \underset{\text{homeo}}{\approx} X_K \right)$$

Remark $\quad$ K: fibered knot のとき
X: symplectic
$\cup$
T symplectically embedded

$\Longrightarrow X_K$ symplectic

Remark' $\quad a_d \neq \pm 1$ のとき $\xrightarrow[\text{Taubes}]{} X_K$ non-symplectic

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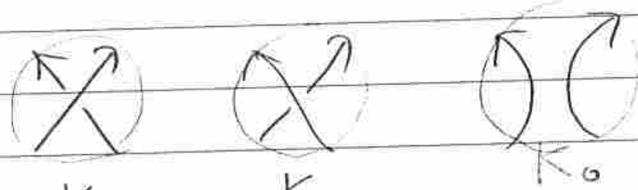
$\Delta_K(t)$



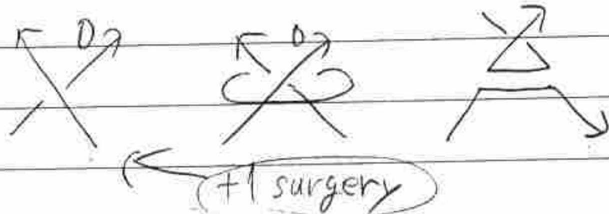
K_+ K_-

$$
* \begin{cases} \Delta_{K_+}(t) = \Delta_{K_-}(t) + (t^{\frac{1}{2}} - t^{-\frac{1}{2}})\Delta_{K_0}(t) \\ \Delta_{\text{trivial}}(t) = 1 \quad \Delta_{\text{split link}}(t) = 0 \end{cases}
$$

X_{K_+} X_{K_-} X_{K_0}

$SW_{X_{K_+}}$ $SW_{X_{K_-}}$ $SW_{X_{K_0}}$

の間の＊に相当する relations



$X_K = X \#_{T = m \times S^1} (K の 0\text{-surgery}) \times S^1$

ThA で $\gamma = pa + qb + rc$

$p=1$ $q=1$ $r=0$

basic class の数が制約される

$L \cdot T = 0$ $T \cdot T = 0$